Journal of Environmental Protection and Ecology 3, No 3, 762–767 (2002)

Clean technologies

TWO PHASE FLOW IN REPLENISHMENT OF GROUND WATER THROUGH HOMOGENEOUS POROUS MEDIA

R. C. SHAH

Department of Mathematics, Nirma Institute of Technology, Sarkhej-Gandhinagar Highway, 382 481 Ahmedabad, Gujarat State, India

Abstract. The phenomenon of fingering and imbibition referred as fingero-imbibition phenomenon for groundwater replenishment, under suitable set of initial and boundary conditions in homogeneous porous media is numerically discussed for its significance. A finite difference method has been employed to solve the governing nonlinear partial differential equation and the numerical results obtained indicate the behaviour of the problem and stabilisation of fingers. These types of problems appear mainly in reservoir engineering, geophysics, geohydrology, etc.

Keywords: porous media, finite difference method, ground water.

AIMS AND BACKGROUND

Is is a well known physical fact that fingering phenomenon arises because of the difference in the viscosities of the two flowing phases whereas imbibition phenomenon is due to wetting difference[1,2]. The phenomena of fingering and imbibition whether occurring singly or simultaneously in the displacement process have gained much importance particularly in reservoir engineering, geophysics, geohydrology, etc. and many authors have studied them by various aspects[3-5] .

The present paper numerically discusses the fingero-imbibition phenomenon for artificial replenishment of ground water in homogeneous porous media with a suitable set of initial and boundary conditions.

MATHEMATICAL MODEL OF THE PROBLEM

Consider that a finite cylindrical piece of porous matrix containing native water (n), is completely surrounded by an impermeable surface except for one end of the cylinder and this end is exposed to an adjacent formation of injected water (i). It is assumed that the injected water and the native water are two immiscible liquids of different salinity with a small viscosity difference; the former represents the preferentially wetting and less viscous phase. This arrangement describes a one-dimensional phenomenon of fingero-imbibition in which the injection of water (i) into the porous medium takes place due to imbibition and consequent displacement of native water (n) produces fingers.

The above problem appears in encroachment of salt water into coastal aquifer containing fresh water, injection of fresh water into air filled porous medium, etc.

For the mathematical formulation, it is assumed that the Darcy's law is valid for the investigated case and also the macroscopic behaviour of fingers governed by a statistical treatment is considered [6].

The basic flow equations governing the phenomenon are as follows[1]:

$$V_i = -\frac{K_i}{\delta_i} K \frac{\partial p_i}{\partial x}, V_n = -\frac{K_n}{\delta_n} K \frac{\partial p_n}{\partial x}, \tag{1}$$

$$P\frac{\partial S_i}{\partial t} + \frac{\partial V_i}{\partial x} = 0, \tag{2}$$

$$V_i + V_n = 0, p_c = p_n - p_i \tag{3}$$

Combining equations (1)-(3) and then using the relations[1].

$$\frac{K_i K_n}{K_i \delta_n + K_n \delta_i} \approx \frac{K_n}{\delta_n}, K_i = S_i, K_n = 1 - S_i, p_c = -\beta S_i$$

yields

$$P\frac{\partial S_i}{\partial t} + \frac{K}{\delta_n}\frac{\partial}{\partial x}\left[(1 - S_i)\frac{\partial}{\partial x}(-\beta S_i)\right] = 0 \tag{4}$$

This is nonlinear partial differential equation governing the phenomenon. A suitable set of initial and boundary conditions becomes

$$\begin{aligned} &S_i(x,0) = S_0; \quad 0 \le x \le L \\ &S_i(0,t) = S_1; \quad t \ge 0 \\ &\frac{\partial}{\partial x} S_i(L,t) = 0; \quad t \ge 0 \end{aligned} \tag{5}$$

where the second condition states that S_1 is the imbibition face saturation and third condition specifies that there is no flow across the face $x = L$.

Substituting

$$1 - S_i(x,t) = S_i^*(x^*, t^*)$$

and then dropping the asteriks, equation (4) with conditions (5) by setting

$$X = \frac{x}{L}, T = \frac{\beta K t}{\delta_n L^2 P}, S_i(x,t) = S_i(X,T)$$

becomes

$$\frac{\partial S_i}{\partial T}-\frac{\partial}{\partial X}\left(S_i\frac{\partial S_i}{\partial X}\right)=0 \tag{6}$$

and

$$\begin{aligned}&S_i(X,0)=1-S_0;\quad 0\le X\le 1\\ &S_i(0,T)=1-S_1;\quad T\ge 0\\ &\frac{\partial}{\partial X}S_i(1,T)=0;\quad T\ge 0\end{aligned} \tag{7}$$

SOLUTION

The finite difference discretisation for equation (6) with the conditions (7) obtained by the Crank Nicolson technique is as follows:

For $i = 1$:

$$\left[-\frac{1}{(\Delta X)^2}\left(S_{i_{2,n+1/2}}+S_{i_{1,n+1/2}}+4\phi\right)+\frac{4}{\Delta T}\right]S_{i_{1,n+1}}+\frac{1}{(\Delta X)^2}\left(S_{i_{2,n+1/2}}+S_{i_{1,n+1/2}}\right)S_{i_{2,n+1}}$$

$$=-\left[\frac{4}{\Delta T}-\frac{1}{(\Delta X)^2}\left(S_{i_{2,n+1/2}}+S_{i_{1,n+1/2}}+4\phi\right)\right]S_{i_{1,n}}-\frac{1}{(\Delta X)^2}\left(S_{i_{2,n+1/2}}+S_{i_{1,n+1/2}}\right)S_{i_{2,n}}-\frac{8\phi^2}{(\Delta X)^2}$$

with

$$S_{i_{1,n+1/2}}=S_{i_{1,n}}-\frac{\Delta T}{2}\left[\left(\frac{S_{i_{2,n}}-3S_{i_{1,n}}+2\phi}{(\Delta X)^2}\right)S_{i_{1,n}}+\left(\frac{S_{i_{2,n}}+S_{i_{1,n}}-2\phi}{2(\Delta X)}\right)^2\right]$$

For $2\le i\le R-1$:

$$\frac{1}{(\Delta X)^2}\left(S_{i_{i-1,n+1/2}}+S_{i_{i,n+1/2}}\right)S_{i_{i-1,n+1}}-\left[\frac{1}{(\Delta X)^2}\left(S_{i_{i+1,n+1/2}}+S_{i_{i-1,n+1/2}}+2S_{i_{i,n+1/2}}\right)+\frac{4}{\Delta T}\right]S_{i_{i,n+1}}$$

$$+\frac{1}{(\Delta X)^2}\left(S_{i_{i+1,n+1/2}}+S_{i_{i,n+1/2}}\right)S_{i_{i+1,n+1}}=-\frac{1}{(\Delta X)^2}\left(S_{i_{i-1,n+1/2}}+S_{i_{i,n+1/2}}\right)S_{i_{i-1,n}}$$
$$-\left[\frac{4}{\Delta T}-\frac{1}{(\Delta X)^2}\left(S_{i_{i+1,n+1/2}}+S_{i_{i-1,n+1/2}}+2S_{i_{i,n+1/2}}\right)\right]S_{i_{i,n}}-\frac{1}{(\Delta X)^2}\left(S_{i_{i+1,n+1/2}}+S_{i_{i,n+1/2}}\right)S_{i_{i+1,n}}$$

with

$$S_{i,n+1/2} = S_{i,n} + \frac{\Delta T}{2}\left[\left(\frac{S_{i-1,n} - 2S_{i,n} + S_{i+1,n}}{(\Delta X)^2}\right)S_{i,n} + \left(\frac{S_{i+1,n} + S_{i-1,n}}{2(\Delta X)}\right)^2\right]$$

For $i = R$:

$$\frac{1}{(\Delta X)^2}\left(S_{i_{R-1,n+1/2}} + S_{i_{R,n+1/2}}\right)S_{i_{R-1,n+1}} - \left[\frac{1}{(\Delta X)^2}\left(S_{i_{R-1,n+1/2}} + S_{i_{R,n+1/2}}\right) + \frac{4}{\Delta T}\right]S_{i_{R,n+1}}$$
$$= -\frac{1}{(\Delta X)^2}\left(S_{i_{R-1,n+1/2}} + S_{i_{R,n+1/2}}\right)S_{i_{R-1,n}} - \left[\frac{4}{\Delta T} - \frac{1}{(\Delta X)^2}\left(S_{i_{R-1,n+1/2}} + S_{i_{R,n+1/2}}\right)\right]S_{i_{R,n}}$$

with

$$S_{i_{R,n+1/2}} = S_{i_{R,n}} + \frac{\Delta T}{2}\left[\left(\frac{S_{i_{R-1,n}} - S_{i_{R,n}}}{(\Delta X)^2}\right)S_{i_{R,n}} + \left(\frac{S_{i_{R,n}} - S_{i_{R-1,n}}}{2(\Delta X)}\right)^2\right]$$

The numerical results obtained are as shown in Table 1 for ΔT = 0.005 s, $S_0 = 0.0, S_1 = 0.5$.

Table 1. Numerical results for the fingero-imbibition phenomenon through homogeneous porous media

X	$T = 0.005$ s	$T = 0.010$ s	$T = 0.015$ s	$T = 0.020$ s	$T = 0.025$ s
0.05	1.735196E–1	2.496144E-1	2.930865E-1	3.213326E-1	3.412201E-1
0.15	2.742128E-2	7.627088E-2	1.160065E-1	1.472597E-1	1.722413E-1
0.25	4.704747E-3	1.931006E-2	3.990790E-2	6.072330E-2	8.000869E-2
0.35	8.072070E-4	4.437567E-3	1.192046E-2	2.214639E-2	3.355816E-2
0.45	1.384948E-4	9.565750E-4	3.202366E-3	7.193364E-3	1.265925E-2
0.55	2.376195E-5	1.977032E-4	7.963793E-4	2.122590E-3	4.326825E-3
0.65	4.076924E-6	3.968560E-5	1.872197E-4	5.806449E-4	1.358392E-3
0.75	6.995906E-7	7.799764E-6	4.221840E-5	1.497994E-4	3.977940E-4
0.85	1.206190E-7	1.516418E-6	9.296299E-6	3.729990E-5	1.115270E-4
0.95	2.412381E-8	3.338012E-7	2.393315E-6	1.080160E-5	3.622084E-5

RESULTS AND DISCUSSION

From the graphs it is observed that the saturation has a decreasing tendency along the space coordinates whereas it has increasing tendency along the time (Figs 1 and 2). Also from Fig. 1 it is seen that for all values of T, the rate of decrease in saturation of injected phase reduced higher value to comparatively lower one as passes through from X = 0.15. At X = 0.055, the saturation almost tending to zero which indicates that the porous medium may be expected to be fully saturated at this point.

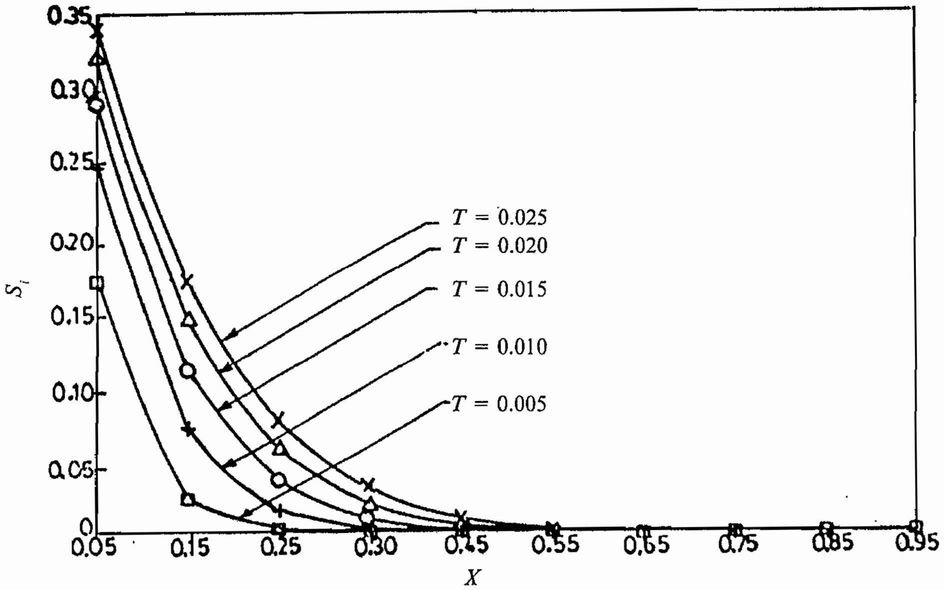

Fig. 1. Space dependent dislacing phase saturation

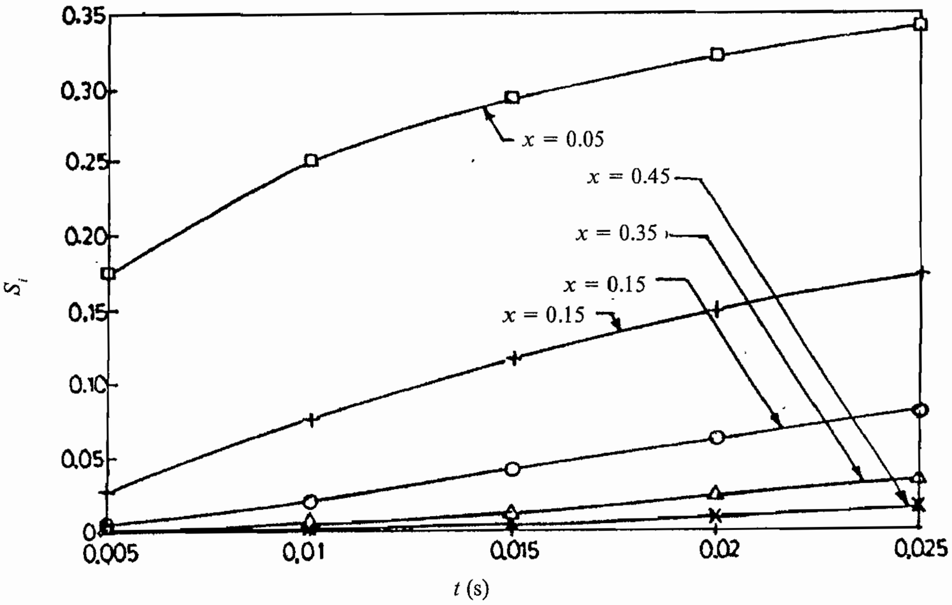

Fig. 2. Time dependent displacing phase saturation

Thus, the present study numerically discusses the fingero-imbibition phenomenon in a displacement process involving two immiscible liquids of different salinity through homogeneous porous media. A finite difference method has been employed to solve the governing nonlinear partial differential equation under some suitable initial and boundary conditions. Since the saturation has been defined as the average cross-sectional area defined by the fingers, i.e. macroscopic behaviour of fingers governed by statistical treatment, $S_i \rightarrow 0$ (decreasing tendency of saturation) along the space coordinates may be considered as a criterion for investigating the stabilisation of fingers[7] .The graphs of the numerical results show the decreasing nature of the saturation along the space coordinates.

On the basis of the present study, it has been concluded that under the consideration of the specific boundary conditions and the values of the various parameters, the stabilisation of fingers is truly possible.

NOMENCLATURE

V_i, V_n – filtration velocity of injected, native liquid (m/s)
K – permeability of the porous media (m^2)
K_i, K_n – fictitious relative permeability of injected, native liquid
p_i, p_n – pressure of injected, native liquid (kg/ms^2)
P – porosity of the medium
t – time (s)
x – linear coordinate (m)
β – capillary pressure coefficient (N/m^2)
δ_i, δ_n – viscosity of injected, native liquid
S_i, S_n – saturation of injected, native liquid
l – length of the porous medium
ϕ – initial saturation at the imbibition phase $(1 - S_1)$

REFERENCES

1. A. P. VERMA: Water Resources Research, **6**, 906 (1970).
2. R. C. SHAH: J. Inst. of Eng., CH, **81**, 21 (2000).
3. A. P. VERMA: J. Appl. Phys., **41**, 3638 (1970).
4. R. J. M.DE WIEST: Geohydrology, John Wiley, New York, 1967.
5. M. N. MEHTA: J. Indian Academy of Math., **16** (1994).
6. A. E. SCHEIDEGGER, E. F. JOHNSON: Can. J. Phys., **39**, 326 (1961).
7. A. P. VERMA: In Multiphase Transport Fundamental Reactor Safely Application, Hemisphere Public, Washington, 1980.

Received 22 April 2002
Revised 20 July 2002