

MATHEMATICAL MODELLING OF PARAMETERS CONNECTED WITH MOISTURE IN BUILDINGS WALLS

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Abstract. In this paper we approach the capillary problems. The capillary phenomena work for the tubes with a diameter less than 2–3 mm. The capillary water is affected by the superficial pressure and that of the gravity field. The capillaries have irregular forms and dimensions in the foundation area. The water ascends in them, preferring the channels with smaller diameters and closing the air where the diameters are bigger. Because of the interaction forces between the water and the solid skeleton a tension state appears in the water from the pores, which, if we consider that the gravitational potential does not exist, is equal to the suction effect. We can deduce from there also the fact that the suction is the main factor that conditions the water migration through porous bodies only when the effect of the gravity potential can be ignored.

Keywords: mathematical modelling, parameters, moisture in buildings walls.

AIMS AND BACKGROUND

Generally, a building must hold to water infiltrations in any of its parts, including foundations and walls. Among the various kinds of building moisture, there is a special one which originates in the water on which the base of structure was constructed. If the phenomenon is manifested by water migration through the base of structure, the building basement or walls, the materials humidity increases above 10–25% and is called moisture¹.

Such a high moisture content in construction materials has two negative consequences:

- it influences the material strength and endurance, together with other factors such as freezing-de-freezing, chemical substances and so on, with the most frequent impact on old monuments and buildings;

- it influences regular working conditions for buildings, as moisture creates an unpleasant inside atmosphere, which most frequently impacts on regular buildings. Moisture can affect insulation in time, especially if there are construction mistakes or if waterproofing measures lack. In all circumstances, the first necessary action is to find the real cause of moisture and then find the most efficient measures to remove it.

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The following simplifying hypotheses lie at the basis of the study²:

– Due to the interaction forces between water and the solid frame, there is a state of stress, which equals suction if the gravity potential is neglected. Hence the conclusion that suction is the main factor influencing water migration through porous bodies. The transport potential of water through porous materials when the effect of gravity can be disregarded is the water pressure through the pores. In case porous materials are not saturated and the effect of overload on water pressure in the pores, the transport potential equals transposed suction ($u = -s$).

– Suction use is limited to porous earths or materials, which do not have an extensive content of salts dissolved in the pore water. If there are substances dissolved in water, the earth pores begin to feel the influence of osmotic pressure and the concept of ‘suction’ must be generalised, meaning that it must include the effect of all the forces determining the state of water in the pores.

– As suction is a manifestation of the interaction between water and the solid frame of the porous material, it will be influenced by the same factors that determine interaction, that is:

- nature and composition of the solid frame and of the liquid;
- the liquid content;
- temperature;
- electrical state;
- salt content, etc.

If only moisture is considered to be varying, and the other factors mentioned do not manifest their influence, then a variation curve of suction and moisture will be determined for each porous material. The curve rate will depend on the type of material.

EXPERIMENTAL

The basic physical analysis of moisture shows that this phenomenon is a dynamic process consisting in a continuous water migration in few of the areas in the soil, and in guiding through the base and into the higher parts of the buildings, where water migration is lost by evaporation into the atmosphere. This water transport is produced by a few parameters, such as:

- water pressure in the pores (u);
- negative pressure or suction (s);
- temperature;
- pressure of water vapours;
- osmotic pressure;
- electric potential, etc.

Of all these parameters, the most important are water pressure in the pores u and suction s ; however, in certain conditions, other parameters may become significant as well.

Usually, migration of water through a non-saturated porous material can be represented by the Darcy law, generalised in the form proposed by Richards³:

$$v = k_w du/dl \quad (1)$$

where v is the flux (speed) of water migration; k_w – the hydroconductivity coefficient for the non-saturated porous material (the water quantity that passes in a time unit through the surface unit expressed in cm/s); du – the pressure difference of water in the pores between two points (water pressure in the pores expressed as height of water column); dl – moisture difference between the same points (length of distance expressed in cm).

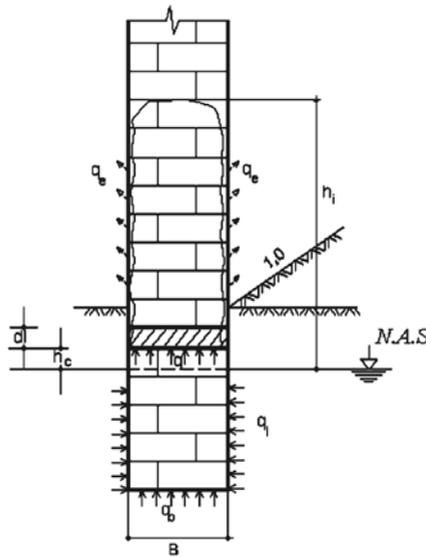


Fig. 1. Schematic water movement in a wall affected by moisture

The water quantity migrating at the base of a brick wall with width B and height dl is given by relation (2). The hydroconductivity coefficient k_w and du are considered as constant, and water migration going through the bottom side of dl (see Fig. 1) will be:

$$q = vB = Bk_w du/dl \quad (2)$$

In order to calculate the values of q , the du/dl ratio and the k_w coefficient in relationship (2) will have to be determined.

RESULTS AND DISCUSSION

Determining the pore water pressure difference between two points. u can be determined beginning with the variation law of u , for a porous material which according to Silvan can be written as follows:

$$u = a_{\text{cmp}} p - s \quad (3)$$

where a_{cmp} is the compressibility factor determined through experiments; p – total pressure in a porous medium; s – suction in the respective area (noticed).

The compressibility factor has values depending on the nature of the material under study:

- for saturated fat clays $a_{\text{cmp}} = 1$;
- for practically incompressible masonry, we have $a_{\text{cmp}} = 0$;
- for the rest of earths (including unsaturated fat clays) $0 < a_{\text{cmp}} < 1$.

Consequently, we will have:

$$u = -s. \quad (4)$$

In the case of earth with a practically incompressible frame, or of the material with a rigid structure, the problem is simplified due to the fact that water in the pores does not take part of the overload and pressure p is fully supported by the solid frame. If we take into account that the origin of water pressure in pores u is atmospheric pressure (meaning the pressure corresponding to the level of underground waters) as well as the definition of suction s as the decrease of water pressure in the ground below atmospheric pressure, it results that both have the same origin but opposite directions.

Pressure u of pore water can be established by taking into account the level of underground water, suction s of the considered element results. Water pressure in the pores is considered to be null when it equals atmospheric pressure and this requirement is met above a layer of underground water with a free level⁴.

Suction equals in numbers the water pressure in the pores. As it depends on the height above the level of underground waters, it means that moisture at any point above the level of underground waters can be directly established from the relation: (pF , w).

As the range of moisture suctions is very wide (from zero to tens of thousands of kg/cm^2) Schofield suggested that in analogy with the scale of acidity pH, the Sørensen pF scale should be used to express suctions on a logarithmic scale (often named improperly a humidity potential or capillary) the decimal logarithm of suction, expressed in centimetres of water column. Thus, if the suction of a material is H_{cm} water, then:

$$pF = \lg H. \quad (5)$$

The symbol p shows its logarithmic character, while F seeks to remind that by defining pF as the logarithm of height in centimetres of the water column corresponding to suction, the ‘free enthalpy’ difference logarithm is used, measured on the gravity scale, as we shall show next. De Bruijn uses for pF the very appropriate term of ‘adsorption exponent’.

For earths, suction can vary usually between the following limits:

- zero, for the case of the saturated material in contact with free water ($H=0$, $pF = -\infty$); usually in practice, the value $pF = -\infty$ is eliminated considering as the lower limit the suction corresponding to a centimetre of water column ($H=1 \text{ cm} = 10^0$, $pF=0$);

- 10 000 of kgf/cm^2 (approximate value) for the case when the material is completely dry in oven at 105°C ($H=10^7\text{cm}$, $pF=7$).

Using suction in the way shown so far is limited to porous earths or materials that do not have a high salt content dissolved in pore water².

For an unitary treatment of water circulation problems below the level of underground waters (positive) or above (negative), both the positive pressure of pore water and the negative pressure should be expressed in the height of water column; in the first case – positive, in the second – negative.

Also, suction can be determined by experiments depending on moisture (w) and on the type of material with the relation:

$$s = f(w, \text{type of material}) \tag{6}$$

whose qualitative form is presented in Fig. 2.

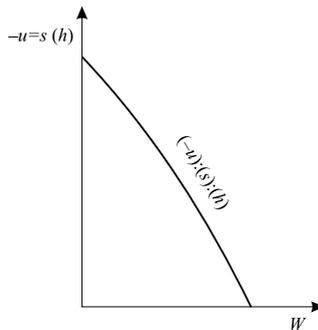


Fig. 2. Quality correlation between suction and humidity

The chart (Fig. 2) proves that the earth suction increases with the decrease of moisture. These suction increases are even higher as humidity is lower. The suction-humidity curves (s,w) are used to track the influence of various factors on the ability of the material to allow water in or out.

A differenced expression (4) results in:

$$du = -ds. \tag{7}$$

The ds variation of suction s along the distance dl , will be produced by the moisture variation in the same interval dl . Suction s identical to the pressure of water in the pores u , inside the capillary area h_p , as function of the capillary height h_c , must be a rising curve. It begins from value 0 at the level of water in the soil (N.A.S.) to the maximum value at h_p , as in Fig. 3, where the curve shape is presented as a linear curve only qualitatively⁵. This law, correlated with relation (6) or Fig. 2, actually indicates a decrease of moisture w along the height h_1 and is presented only qualitatively in Fig. 3.

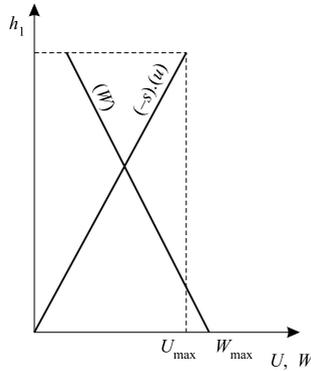


Fig. 3. Quality correlation between the pressure of water in the pores inside the capillary area as function of the capillary height

Thus, we can have:

$$u = K h_c, \quad (8)$$

where K is constant.

It results:

$$du/dl = du/dh_c = K \quad (9)$$

where du is the pressure difference of pore water between two points; dl – the moisture difference between the same points; dh_c – the capillary height difference.

Thus, relationship du/dl in a first approximation can be considered to be constant depending on w , as represented qualitatively by curve a in Fig. 4.

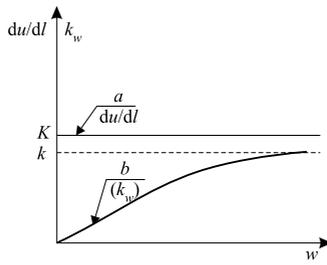


Fig. 4. Quality dependence of moisture for dampness of walls parameters

Determining the hydroconductivity coefficient. k_w can be determined experimentally and it can be approximated by a law indicated by Gardner² as follows:

$$k_w = a/(s^m + b) \quad (10)$$

where s is suction; a and b – constant; m – an exponent with values between 1 and 4, except the sands for which there are higher values.

The a/b relationship equals the permeability in a saturated state ($s=0$).

For the quantity characterisation of the capacity to guide water, k_w is used, which is not a constant but it depends on the moisture state as it is the physical characteristic of the body that shows its ability to be filled with water⁶. The quality variation of k_w is presented in Fig. 5.

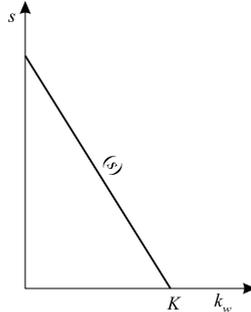


Fig. 5. Quality variation of k_w

From Fig. 5 and the results obtained by various authors who used these methods to determine the hydroconductivity coefficient, it results that its values are much influenced by moisture, in the sense that while moisture is reduced, the hydroconductivity coefficient is extremely.

According to Gardner, the influence of suction is not only manifested on water pressure in the pores of unsaturated materials (and implicitly on the total potential that conditions water movement) but it directly conditions the value of the hydroconductivity coefficient (relation (10) – Fig. 5), in the sense that by increasing suction, we reach important decreases of the hydroconductivity coefficient.

In order to obtain the manner of variation between k_w and w , we can use the interpretations represented in Figs 2 and 5. Hence, k_w will have a rising shape and a horizontal asymptote, which corresponds to the saturation value (k permeability coefficient). As long as the considered material is saturated and it maintains its porosity, its moisture, its salt content, etc., the hydroconductivity coefficient generally has a constant value for the determination of which the concept of ‘permeability coefficient’ or ‘filtration’ became commonly used in literature and noted with k . The methods to determine the permeability coefficient are given in different works that deal with water movement in saturated porous media. To determine the permeability coefficient, the method proposed by Moraru will be used and namely: under the action of pressure, a fluid passes through a porous medium with a certain flow. This is given by the following expression:

$$Q = (k/\eta) S (P/l) \quad (11)$$

deduced from the Darcy law combined with the Poiseuille law in which P is pressure; S – crossing section; k – permeability of the porous medium; η – dynamic viscosity; l – distance length.

We begin with the Darcy law:

$$V = ki = k (p/l) \quad (12)$$

and the Poiseuille law:

$$Q = \frac{\pi r_i^2 P}{8 \eta l} \quad (13)$$

where r_i is the average capillary radius.

The current speed was outlined to identify it to that of the Darcy formula. We obtain the expression of the permeability coefficient in relation (11):

$$\frac{P r^2}{8 \eta l} = k \frac{P}{l} \quad (14)$$

$$k = \frac{r^2}{8 \eta} \quad (15)$$

where r is pore radius (cm); η – dynamic viscosity of liquid (Poise).

Returning to the variation curve for the hydroconductivity coefficient k_w with the moisture w , in Fig. 4. is presented qualitatively by curve b .

Determining the water quantity that can be evacuated through the contact area with air through evaporation. In order to ensure the stationary character of the phenomenon, migration q that enters through the lower area of the wall equals migration q_e , which can be evacuated through the contact area with air through

vaporisation. This migration, according to Moraru and Valcea can be written as follows⁵:

$$q_e = 2h_1 l e \quad (16)$$

where l is length of the studied wall (1.0 m); e – wall vaporisation coefficient (g/day m²), depending on the material properties, temperature and ventilation, etc.

The quality aspect of a presented problem is enough to find the practical measures necessary to decrease moisture.

CONCLUSIONS

Water migration which enters the lower area of the wall is approximately the same with the water which is vaporised on the two side surfaces of the wall.

The pore water pressure is considered void when it is equal with atmospheric pressure. This condition is achieved at the top of a layer of underground water level freely.

Hence the idea that ventilation methods used in buildings that can not suffer another intervention will have the expected effect.

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